Mass-customization of insurance: A valuation perspective

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Abstract

The insurance industry faces new challenges of changing demographics, volatile financial markets, disruptive technology, and intensifying competition. The recent COVID-19 pandemic might increase the need and urgency to develop new solutions and adopt new practices. This paper presents a large set of flexible products that can be easily tailored to meet individual customer demand, using mass-customization and technology enabled personalization. These are structured products that combine components of saving and insurance, which include an upside linked to a market index on the one hand, and a downside protection on the other hand. We define the fair valuation as the expectations taken under the physical measure discounted by the risk-free rate. We focus on a detailed presentation and discussion of three families of such products and their valuation, first in a general formula, later by mathematically solving typical examples, and lastly by proving numerical examples.

Keywords: *insurance products; life insurance; mass-customization; structured-products*

1. Introduction

Over the past 50 years, equity-linked contracts with death and survival benefits have been studied in the literature and applied in the insurance industry. The first saving contract is a well-known guaranteed minimum maturity benefit studied by Brennan and Schwartz (1976) and later by Bacinello and Ortu (1993). In recent years, there is a shift into more flexible insurance products (see, Bacinello et al. (2011)). The paper is motivated by the challenges facing the insurance industry in our era. These include the insurance competitive nature, the customers' demographics and tastes, and the financial markets' characteristics. We mention here only a few references for these challenges. Quoting McKinsey's Segev and Vickers (2017): "From a customer's standpoint, few industries are as ripe for disruption as insurance." Summarizing trends of life insurance in developed countries, mainly Europe, North America, and Japan, Strovink et al. (2018) conclude that "longer lifetimes will require products that provide guaranteed income in retirement to supplement pensions." The financial markets do not follow historical patterns, as advocated by renowned economist Mohamed El Erian (2010), who argues that we have entered "a new normal" where the global economic landscape has dramatically changed, and interest rates and investment returns that we saw before the global crisis are converging to lower long-term averages. This notion is supported also by others, such as Summers (2013; 2014), who predicts long term stagnation and turbulence in financial markets. The new environment requires new products under different assumptions.

It seems inevitable that competition would compel insurers to offer customized flexible product to their clientele, and that such offering would have to be performed at a low cost to a large population. Personalization is at the forefront, especially since the internet boom some 20 years ago, see for example Adamova et al. (2018). However, the personalization notion has older roots in the idea of mass customization, see for example Pine, Victor & Boynton (1993) and Anderson & Pine (1997). We think that the products we present in this paper effectively address these challenges and are likely to be widely adopted in insurers offering. Thus, their proper and quick valuation is necessary to enable practical applications.

For a reader unfamiliar with structured products we start with a brief introduction. Structured products are pre-packaged investment assets, typically combining characteristics of bonds and derivatives, generating a desired payoff scheme. The payoffs, defined by contracts, are usually contingent on one or a few underlying assets and are practically unlimited in their functional form. An investor wishes to gain high positive returns without exposing her invested fund to the risks of negative returns associated with common assets such as equity. A typical contract promises investors the higher of either the future value of the fund invested in the risk-free interest rate or the future value of the fund invested in a basket (portfolio) of risky assets. For example, a basket of risky assets may include a combination of a few indices, e.g. 50% in S&P-

500, 25% in DJ Eurostoxx-50 and the remaining 25% in KOSPI-200. In Section 2 we present a much wider spectrum of structured products.

Structured products have attracted the research community attention. Examples start decades ago, e.g. Bitran & Pedrosa (1998) and continue more recently with Cox, Fairchild & Pedersen (2004), Benet, Giannetti & Pissaris (2006), Yosef (2006), Hens & Rieger (2008) and others. Earlier research on structured products in the context of the insurance industry focus on the downside protection for investor, these include Cox et al. (2004) and Yosef (2006) for example. More recent examples of structured product valuation in the insurance context include Maegebier (2013), Dickson et al. (2013), Jang and Ramli (2015), Gerber et al. (2015), Zhou and Wu (2015), Cui et al. (2017), and Chong (2019). Our work continues this strand of literature and we believe it is the first to provide a broader and general view of the flexibility structured products offer insurers and insureds on the one hand, and the valuation scheme for their practical application on the other hand. Our paper relevance rides on global trends and anticipated needs encompassing the changing demographics, longevity, financial market regimes, competitive forces, and technology.

There are virtually endless examples and ways to personalize such products. We present three architypes as examples. For each architype there are numerous applicable specifications. We choose a few practical specifications, with two stochastic processes underlying a market index, and a few interest rates, for a rich set of numerical examples that can fit the limited scope of a single paper.

The remainder of the paper is organized as follows. Section 2 presents the general model, Section 3 provides detailed solution examples, Section 4 discusses the results, and Section 5 concludes.

2. The general model

We start with a simple example of a saving product, then we generalize it. Later we present a hybrid of life insurance and saving product followed by a hybrid of disability insurance and saving product. We focus here on presenting the general ideas and models and leave closed-form solutions and numerical calculations to later chapters.

2.1 A saving product

As an opening example for the more general model, assume an insurer offers a saver a saving plan in which the saver invests B dollars for a period t^0 in a product that offers potential positive returns and includes protection against losses on the investment. The product (i.e. the saving plan) pays only if the saver is alive throughout the period t^0. If at t^0 the saver is not alive the contract is void and the product pays nothing.[1] More specifically, assume that by the end of the period, at t^0, the saver is alive, then the insurer pays her an amount $B(1-k)\cdot R_I\left(t^0\right)$ if $R_I\left(t^0\right)>1$ and $B(1-k)$ otherwise, where $R_I\left(t^0\right)$ is the gross return (by the end of the period t^0) of a benchmark index or portfolio (basket) specified in the contract, and k is the insurer commission (including all its costs and profit).[2] Actually, this contract is a combination of a Pure Endowment insurance policy and a structured product. An advantage of this contract, in addition to its lower cost compared to that of a classical structured product, could also be a tax relief that is usually provided for insurance policy contracts.

Now, assume that the remaining lifetime of the saver is a random variable T, then her payoffs at the end of the contract can be expressed by:

$$(1)\qquad PO_{BF}\left(t^0\right)=1_{T>t^0}\cdot B(1-k)\cdot\left\{1+\left(R_I\left(t^0\right)-1\right)^+\right\},$$

where: BF denotes the basic floor product,

1_{cond} is the indicator function (it equals 1 when *cond* is true and 0 otherwise),

$(y)^+$ is a short form for max(y, 0),

and the other variables are defined above.

The present value (PV) of this contract (often called *fair value*) is the discounted expectations of the future payoffs in equation (1). We use the common practice of insurance contracts - calculate the expectations under the physical measure and discount

[1] In this paper we use the terms *product* and *contract* interchangeably. In this section we also use the alternative term *saving plan*.

[2] For simplicity and without loss of generalization we assume zero tax effects throughout the paper. Tax regulations and their effect change from jurisdiction to jurisdiction and within a jurisdiction it could depend on personal characteristics such as age, income, etc.

it using the risk-free rate.[3] The expectations are written in equation (2) and their present value in equation (3).

$$(2) \qquad E\left[PO_{BF}\left(t^0\right)\right] = \mathrm{P}\left(T > t^0\right) \cdot B(1-k) \cdot \left\{1 + E\left[\left(R_I\left(t^0\right) - 1\right)^+\right]\right\},$$

where $\mathrm{P}\left(T > t^0\right)$ is the probability that the saver is alive at the end of the period (policy).

$$(3) \qquad PV_{BF}(0) = e^{-rt^0}\,\mathrm{P}\left(T > t^0\right) \cdot B(1-k) \cdot \left\{1 + E\left[\left(R_I\left(t^0\right) - 1\right)^+\right]\right\},$$

where r is the risk-free rate.

Comment: we assume that we can ignore the case of $T = t^0$, that it is of zero measure or, if time is measured by calendar days, the specific contract would specify the payoffs when death and end of contract occur on the same day.

Equation (3) expresses the fair value of the basic floor contract, the price at which an insurance company can offer such a product to its customers. It can be easily calculated using Monte Carlo simulations, and a closed-form solution is feasible only for certain processes of $R_I\left(t^0\right)$. We provide such solutions in the next section.

In the above example there are two simple parameters: (i) the guaranteed floor is the principal, i.e. the saver, if alive on t^0 would receive an amount no less than her principal B (less a commission); (ii) a payoff above the floor would occur when the return on the benchmark index (or portfolio) is positive, i.e. when $R_I\left(t^0\right) > 1$.[4] For various reasons, a saver might wish to invest in a more advanced product, tailored to her specific needs, constraints, and risk tolerance. Equation (4) expresses the payoffs of such a product.

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[3] This applies also for some derivative contracts under certain conditions which apply here. See for example Afik (2015) for more details.

[4] In our example a commission (at a rate k) is deducted from the payoffs in both cases - when $R_I\left(t^0\right) > 1$ and when $R_I\left(t^0\right) \le 1$. However, when it is desirable (or required), it is easy to change the expression and accommodate a specific commission rate for each scenario. For simplicity and without loss of generality we ignore such alternative contract specifications in this paper.

(4) $$PO_{FF\&T}\left(t^0\right)=1_{T>t^0}\cdot B(1-k)\cdot\left\{R_{floor}\left(t^0\right)+\beta\left(R_I\left(t^0\right)-R_{th}\left(t^0\right)\right)^+\right\},$$

where: FF&T denotes a flexible floor and threshold product,

$R_{floor}\left(t^0\right)$ is the floor function, the minimal payoff of each dollar invested,

$R_{th}\left(t^0\right)$ is the threshold function, any excess return of $R_I\left(t^0\right)$ above this threshold would increase the payoff above the floor,

β is the rate at which the excess return affects the payoffs,

and the other variables are defined earlier.

The contingent payoff scheme of the flexible floor and threshold (FF&T) has a few appealing features: (i) it is contingent on $T>t^0$, i.e. the saver is alive throughout the contract period. Though one could regard it as a drawback to the saver, this feature also lowers the price of the product to her. (ii) The floor $R_{floor}\left(t^0\right)$, which sets the minimum payoff level for each dollar invested, allows for flexible customization of the product. In our basic example (BF above) it is set to 1, a perfect protection of the principle B less the insurer commission. Some savers may wish to lower the price of the product and choose $R_{floor}\left(t^0\right)<1$, whereas others, willing to pay a higher price, could require a higher floor, such as the return on a treasury bond e^{rt^0} or another agreed upon floor return. (iii) The threshold return $R_{th}\left(t^0\right)$ allows similar flexibility to that of the floor, yet it affects the price and future payoffs of the saver in an opposite direction compared to the floor defined by $R_{floor}\left(t^0\right)$. (iv) β defines the rate at which the excess return contributes to the future payoffs $\left(R_I\left(t^0\right)-R_{th}\left(t^0\right)\right)^+$. Compared with our BF example (where $\beta=1$) requiring a higher potential future payoff by setting $\beta>1$ increases the price and vice versa.

The present value of the flexible floor and threshold product is then:

(5) $$PV_{FF\&T}(0)=e^{-rt^0}\,\mathrm{P}\left(T>t^0\right)\cdot B(1-k)\cdot E\left\{R_{floor}\left(t^0\right)+\beta\left(R_I\left(t^0\right)-R_{th}\left(t^0\right)\right)^+\right\},$$

which is derived using similar explanation to that of equation (3). Note that here we keep $R_{floor}\left(t^0\right)$ in the expectation operator, thus allowing for the floor to also be a random process. This flexibility is not a superfluous one. For example, it could be used for a *competition product* in which the saver gains from the higher return between two competing investment indices (portfolios). For example, assume that R_I is chosen to be the (gross) return of the DAX index and R_{floor} is the return of the S&P500 index. Then setting $\beta=1$ and $R_{th}=R_{floor}$ provides the required result. If upon reaching the end of the contract the savor is alive, then if $R_{S\&P500}\left(t^0\right)>R_{DAX}\left(t^0\right)$ she would be paid $B(1-k)\cdot R_{S\&P500}\left(t^0\right)$ dollars, alternatively, if $R_{S\&P500}\left(t^0\right)<R_{DAX}\left(t^0\right)$, she would be paid $B(1-k)\cdot R_{DAX}\left(t^0\right)$ dollars. Equation (5) provides the fair value price for this special case too.

2.2 A hybrid of life insurance and saving product

Whereas the prior section presents flexible alternatives for savings with reduced cost to the saver, enabled by the condition $T>t^0$ (the savor is alive at the end of the period), some customers prefer bundling savings and life insurance. Hence we present a product that pays PO_{saving} at t^0 if the insured is alive at the end of the contract (i.e. $T>t^0$), and if the insured dies earlier ($T\le t^0$) the contract pays its estate an amount of PO_{mort} at T. This kind of policy contract is often referred to as an *endowment insurance policy*. The contingent payoffs PO_{saving} and PO_{mort} could be defined as fixed amounts or a function of deterministic and random variables. For example, a flexible contract may include the following saving payoffs $PO_{saving}=R_{floor}\left(t^0\right)+\beta\left(R_I\left(t^0\right)-R_{th}\left(t^0\right)\right)^+$ and/or a similar function for the mortality payoffs.

(6) $$PO_{FI\&S}\left(T\wedge t^0\right)=B(1-k)\cdot\left[1_{T>t^0}\cdot PO_{saving}\left(t^0\right)+1_{T\le t^0}\cdot PO_{mort}\left(T\right)\right],$$

where $T\wedge t^0$ means $\min\left(T,t^0\right)$ and *FL&S* means flexible life and savings.

The present value of the flexible insurance and saving product is:

(7) $$PV_{FL\&S}\left(0\right)=B\left(1-k\right)\cdot E\left[1_{T>t^0}\cdot e^{-rt^0}PO_{saving}\left(t^0\right)+1_{T\le t^0}\cdot e^{-rT}PO_{mort}\left(T\right)\right],$$

Since t^0 is a constant, specified in the contract, equation (7) can be simplified to:

(8) $$PV_{FI\&S}\left(0\right)=B\left(1-k\right)\cdot\left\{\mathrm{P}\left(T>t^0\right)\cdot e^{-rt^0}E\left[PO_{saving}\left(t^0\right)\right]+E\left[1_{T\le t^0}\cdot e^{-rT}PO_{mort}\left(T\right)\right]\right\}$$

which could be further simplified and then solved for certain specifications of the payoffs. We provide such examples in Section 3.

2.3 A hybrid of disability insurance and saving product

Our final example of a customized offering is a disability insurance bundled with saving product. Let us designate the time to a stochastic disability event by T_D, a random variable. On T_D day, if occurs during the contract coverage period ($T_D \le t^0$), the product pays the insured an amount of $PO_{disab}\left(T_D,\omega\right)$, where $\omega\in\left[0,1\right]$ is a random variable defined by the disability level, 0 is no disability and 1 is the highest level of disability. Otherwise, if $T_D>t^0$, the contract pays $PO_{saving}\left(t^0\right)$ on t^0. The future payoffs of this products are mathematically defined by:

(9) $$PO_{FD\&S}\left(T_D\wedge t^0\right)=B\left(1-k\right)\cdot\left[1_{T_D>t^0}\cdot PO_{saving}\left(t^0\right)+1_{T_D\le t^0}\cdot PO_{disab}\left(T_D,\omega\right)\right],$$

where *FD&S* means flexible disability and savings, and its variables are defined above.[5]

The fair present value of this product is:

(10) $$PV_{FD\&S}\left(0\right)=B\left(1-k\right)\cdot E\left[1_{T_D>t^0}\cdot e^{-rt^0}PO_{saving}\left(t^0\right)+1_{T_D\le t^0}\cdot e^{-rT_D}PO_{disab}\left(T_D,\omega\right)\right].$$

[5] To simplify the exposition, we ignore mortality issues in this case. Nevertheless mortality could be added as a third random process affecting the product, or it could be assumed that upon mortality the contract validity continues, paying the insured estate $PO_{saving}\left(t^0\right)$ on t^0, or that the disability term is widen to include mortality with $\omega=1$. Each setting requires further elaboration and analysis which are beyond the scope of this paper.

2.4 Cost and value matters

The above expressions, e.g. equation (10), represent the present value (*PV*) of the future payoffs to the investor (the insured customer), net of the insurer's commission (including all its costs and profit). Hence, the amount the investor needs to pay at t=0 to purchase these products is $PV/(1-k)$, an amount which needs to cover all the insurers costs and profit, where the costs include the future payoffs to the customer, the hedging, operation costs, and profit. The operational matters of the insurer such as hedging and risk management and assessment are beyond the scope of this paper and are left for future research.

3. Solution Examples

In this chapter we provide a few examples of closed-form solutions to the contracts presented previously, using well known computations in stochastic process, and numerical results under specific assumptions. Where a closed-form solution is not developed we present results using Monte Carlo (MC) simulations.

The potential universe of examples is huge. Aiming to provide a wide spectrum of examples on the one hand, and to avoid voluminous tables and data on the other hand, we limit our calculations to a set of parameters and underlying models. For the reference index we use the S&P 500, modeling it with two commonly used stochastic processes (GBM and JD) as detailed in section 3.1.1. For the mortality model we present examples using the widely adopted Gompertz law. For the disability model, we are not aware of a widely accepted industry standard, hence, we devise a model after consulting an expert practitioner in the field. We further limit all our examples to a matrix of two ages (30 and 40 years), three product durations (t^0 = 5, 10, and 20 years), and three risk-free interest-rates (1, 3, and 5%). This matrix of 18 entries is presented for each of the three architype products and each of the two index models in six tables, each divided to six panels, each panel for a set of R_{th} , R_{floor}, β_{saving} , and β_{mort} or β_{disab} (where applicable). For simplicity use the same R_{th} and R_{floor} for the saving and life (or disability) components.

We start by presenting the assumed modeling components, including the returns' models, the life duration model, and the disability probability distributions.

3.1 Stochastic model definitions

In this section we present our assumed stochastic models for the purpose of the solution examples.

3.1.1 Return models

Since often the reference index for the various payoffs is a large equity index, we focus our attention on Geometrical Brownian Motion (GBM) and Jump Diffusion (JD) return modeling. While a closed-form solution is not always feasible, for the products presented in Section 2 we can always find a numerical solution using MC methodology. This enables a rather straightforward extension of our models to bond portfolios, exchange rates, etc.

<u>GBM model</u>

We define the GBM processes by:

$$\frac{dS}{S} = \mu dt + \sigma dW$$

where S is the index value, μ is the drift, σ is the volatility and $\Delta W \sim N(0, \Delta t)$ is the Brownian motion. The GBM process value at the end of a period τ (starting at t=0 with S(0) and ending at τ) is:

$$S(\tau) = S(0) \cdot e^{\left(\mu - 0.5\sigma^2\right)\tau + \sigma W(\tau)}$$

The gross returns of the index during this period can thus expressed by:

(11) $$R(\tau) = e^{\left(\mu - 0.5\sigma^2\right)\tau + \sigma W(\tau)}$$

To complete the model definition, for numerical calculations in general, and in particular for Monte Carlo solutions we need to assume the model parameters μ and σ. For the examples of this paper we set them at 0.0542 and 0.1757 respectively, estimated from S&P 500 index weekly returns during the recent 20-year period of May 1999 to June 2019.

JD model

We adopt the commonly accepted jump diffusion model of equation (12), which admits empirical leptokurtic distributions of many financial assets, including equities, indexes, and foreign exchange rates.[6] Following Glasserman (2004) and others in the path laid out by Merton (1976) we assume:[7]

$$(12)\qquad d\tilde{R}(t) = \mu dt + \sigma dW(t) + dJ(t) \quad ,$$

where

μ is the deterministic drift rate.

σ is the deterministic volatility of the process.

$W(t)$ is the Wiener process.

$J(t)$ is the univariate jump process defined by:

$$(13)\qquad J(t) = \sum_{j=1}^{N(t)} \left(Y_j - 1\right) \quad \text{or} \quad dJ(t) = \left(Y_{N(t)} - 1\right) dN(t)$$

where $Y_1, Y_2, \ldots$ are random variables independent of N(t) which is a counting process. We also assume that W(t) and J(t) are independent. Restricting Y_i to positive values ensures that the gross returns (and thus the risky asset value) remain non-negative which is necessary for our model of long positions in the financial market. The solution of equation (12) is:[8]

$$(14)\qquad \tilde{R}(t) = e^{\left(\mu - 0.5\sigma^2\right)t + \sigma W(t)} \prod_{j=1}^{N(t)} Y_j \ .$$

To progress from equation (14) to a numerical solution we need to define the distributions governing the jump arrival and the jump size. Following the common practice, we assume that $N(t)$ is a homogeneous Poisson process with intensity λ, hence the jumps inter-arrival times $t_{j+1} - t_j$ are independent with exponential

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[6] A more comprehensive model would include also stochastic volatility which captures volatility clustering effects; however, this would require the estimation of additional market variables and calibration which may add rigor to the model, yet might introduce additional uncertainty to the estimation. See for example Wilmot (2009) on model calibration.

[7] See also Brigo et al (2009).

[8] For a detailed development of the following expressions see for example Glasserman (2004).

distribution $P\left(t_{j+1}-t_j\le \Delta t\right)=1-e^{-\lambda\cdot\Delta t}$, $t\ge 0$, where t_j is the arrival time of jump *j*. Adopting the Merton (1976) assumption that the jump size is lognormally distributed, i.e. $Y_j \sim LN(\mu_y,\sigma_y^2)$ leads to a compact solution which allows practical calibration of the model and the simulation of the contracts in this paper using market compatible parameters. The assumption $Y_j \sim LN(\mu_y,\sigma_y^2)$, i.e. $ln(Y_j) \sim N(\mu_y,\sigma_y^2)$, results in the following unconditional distribution of the gross returns:

$$(15)\qquad P\left[\tilde{R}(t)\le x\right]=\sum_{n=0}^{\infty}e^{-\lambda t}\frac{(\lambda t)^n}{n!}F_{n,t}(x)\ ,$$

where $F_{n,t}(x)\sim\ LN\left[\left(\mu-\tfrac{1}{2}\sigma^2\right)t+\mu_y n,\ \sigma^2 t+\sigma_y^2 n\right]$.

Same as for the GBM, we use S&P 500 index weekly returns during the recent 20-year period to estimate the parameters of the JD process (using maximum likelihood): μ = 0.1842, σ = 0.09636, λ = 24.48, μ_y = -0.005753, and σ_y = 0.02838.

3.1.2 Mortality model

We now need a distribution for the probability that the individual survives through the life of the policy contract $\mathrm{P}\left(T>t^0\right)$, where *T* is the random variable of the total lifetime for an individual and t^0 is the duration of the contract. For this paper we adopt the commonly used *Gompertz* law:

$$(16)\qquad \mathrm{P}\left(T>t^0\right)=\exp\left[-\frac{\omega}{\ln c}c^{age}\left(c^{t^0}-1\right)\right]\ ,$$

where *c*, ω are positive constants, *T*, t^0 are defined above, and *age* is the present age of the individual. For the purpose of this paper examples, following Yosef (2006), we use: c = 1.1, $\omega = 10^{-4}$, and ages of 30 and 40 years.

3.1.3 Disability model

We are not aware of a commonly accepted disability model. Based on our "intuition" and consulting with a field-expert we assume that a disability model can be approximated using two distributions. Given a disability event, we assume that the disability level (ω in equation (10)) is uniformly distributed: $\omega \sim U\left[0,1\right]$.

For the timing of the disability event (T_D in equation (10)), after consulting with an experienced practitioner in disability assessment, for our numerical examples we assume that the age dependent disability density function is approximated by the Beta distribution: $T_D^{'} \sim Beta(\alpha,\beta)$, where $T_D^{'}$ is the future age of the individual (number of years from birth to a future day) and thus the time from the initiation of the insurance contract ($t=0$) to a disability event ($t=T_D$) is defined by: $T_D = T_D^{'} - age$, where *age* is the age of the insured at $t=0$. Specifically, we assume that $T_D^{'} \sim Beta(6,1)$ and we convert the native support [0, 1] of the Beta distribution to [20, 100]. Furthermore, we assume that the draws for $T_D^{'}$ and ω are independent. Obviously, the simulations can easily handle other assumed distributions and their respective support.

3.2 The saving product example

In this section we study the saving product of equation (5). We start by developing a closed-form solution under the assumption that $R_I\left(t^0\right)$ is represented by a GBM process and $R_{th}\left(t^0\right)$ is a deterministic value, either an arbitrary constant or the risk-free return. The computations rely on well-known results in the stochastic process. We then provide numerical valuation examples for $PV_{FF\&T}\left(0\right)$ for the GMB process using the closed-form solution and for the JD process using MC simulations.

3.2.1 Closed-form solution for the saving product

We assume $R_{floor}\left(t^0\right)$ is a constant. Its typical values would be the return of a risk-free asset e^{rt^0} , 1, 0.95, 0.9, or even lower values when the insured wishes to lower the cost of the product and is willing to take a higher risk. Similarly, we assume $R_{th}\left(t^0\right)$ is a constant. Its typical values would be 1 or the return of a risk-free asset. In the case of the threshold level, when the insured wishes to lower the cost of the product and is willing to accept lower returns (a smaller "participation" in the upside of the index returns), she would prefer setting the threshold at 1.05, 1.1, etc., or simply accept a lower level of β.

Under the above assumptions we calculate the following expectations:

$$E1 = E\left(R_I\left(t^0\right) - R_{th}\left(t^0\right)\right)^+ = E\left(e^{\left(\mu - 0.5\sigma^2\right)t^0 + \sigma W\left(t^0\right)} - R_{th}\right)^+,$$

where for convenience we omit the time dependence of the threshold return R_{th} (its expected value is a constant). Using an explicit expression for the Brownian motion $W\left(t^0\right) = \varepsilon\sqrt{t^0}$, where $\varepsilon \sim N\left(0,1\right)$, a random draw we have the following integral:

$$E1 = \int 1_{\left\{\varepsilon > \frac{\ln(R_{th}) - (\mu - 0.5\sigma^2)t^0}{\sigma\sqrt{t^0}}\right\}} \left(e^{\left(\mu - 0.5\sigma^2\right)t^0 + \sigma\varepsilon\sqrt{t^0}} - R_{th}\right) dF_\varepsilon \ .$$

For convenience we define:

$$d_1 = \frac{\ln(R_{th}) - (\mu - 0.5\sigma^2)t^0}{\sigma\sqrt{t^0}}$$

and: $\overline{\Phi}(z) = 1 - \Phi(z) = \Phi(-z)$,

where $\Phi(z)$ is the distribution function of Z, a standard normal random variable.

Then, calculating the expectation integral for E1 we find:

$$E1 = e^{\left(\mu - 0.5\sigma^2\right)t^0 + 0.5\sigma^2 t^0}\,\overline{\Phi}(d_1 - \sigma\sqrt{t^0}) - R_{th}\overline{\Phi}(d_1) = e^{\mu t^0}\Phi(\sigma\sqrt{t^0} - d_1) - R_{th}\Phi(-d_1) \ .$$

Inserting E1 in equation (5), our closed-form solution for the *saving structure product* is:

(17) $$PV_{FF\&T}\left(0\right) = e^{-rt^0}\,\mathrm{P}\left(T > t^0\right) \cdot B\left(1-k\right) \cdot \left\{R_{floor} + \beta\left[e^{\mu t^0}\Phi(\sigma\sqrt{t^0} - d_1) - R_{th}\Phi(-d_1)\right]\right\}.$$

3.2.2 Numerical examples for the saving product

As mentioned at the beginning of this chapter, the universe of products and scenarios specifications is limitless. Hence, we focus our examples on a few specifications. Table 1 presents the numerical values calculated using equation (17) for the saving product, assuming the reference index returns follow a GBM process. Each line shows results for a specified risk-free interest-rate. These rates are per annum, continuously compounded values. Age and contract period are specified in years.

Table 1 about here

To calculate the present value for the saving product, assuming the reference index returns follow a JD process we use MC simulations. Such simulations provide an approximation to the expected value of $PV_{FF\&T}(0)$ in equation (5). The uncertainty of the MC method result (its confidence interval) diminishes as the number of simulation paths increases. We empirically assess the result uncertainty by running 10,000 paths to form an "observation" of $PV_{FF\&T}(0)$ estimation and we repeat the process 100 times. For example, for *age* = 40 years, R_{th} = 1, R_{floor} = 1, and β = 0.5 we find that the standard deviation of the results is about 0.48% of the average value. Hence, we decide to use 100,000 paths per MC estimation, reducing the standard deviation to about 0.16% of the average value, a level which we consider reasonable for the purpose of the numerical examples in this paper. Table 2 presents the numerical values for the saving product, assuming the reference index returns follow a JD process.

Table 2 about here

3.3 The life insurance and saving product example

In this section we study the life insurance and saving product of equation (8). We follow the process of section 3.2, developing first a closed-form solution under similar assumptions, and then provide numerical valuation examples for $PV_{FL\&S}(0)$ for the GMB process using the closed-form solution and for the JD process using MC simulations.

3.3.1 Closed-form solution for the life insurance and saving product

To develop a closed-form solution we need to specify the payoff functions. We use again the same payoff function of section 3.2, this time for the savings and for the life components:

(18) $$PO_j = R_{floor}^j\left(t^j\right) + \beta^j\left(R_I^j\left(t^j\right) - R_{th}^j\left(t^j\right)\right)^+$$

where j is either *saving* or *mort* and t^j is t^0 or T respectively. $R_I\left(t^j\right)$ for the savings and for the life components are represented by GBM processes, $R_{th}\left(t^j\right)$ are either arbitrary constants or the risk-free returns (which is a stochastic value for the case of *mort*, the life component), and we also assume that $R_{floor}\left(t^j\right)$ are constants. The various parameters and GBM processes are not necessarily the same for the life and saving components, they could be chosen separately to tailor the product for the specific customer's needs.

The saving component definition is identical to that of section 3.2 and thus its value is expressed in equation (17). Hence, in this section we focus our attention on the life insurance component:

$$(19)\qquad PV_{mort} = E\left[1_{T\le t^0}\cdot e^{-rT}PO_{mort}\left(T\right)\right] = E\left\{1_{T\le t^0}\cdot e^{-rT}\left[R_{floor}^{mort}\left(T\right)+\beta^{mort}\left(R_I^{mort}\left(T\right)-R_{th}^{mort}\left(T\right)\right)^+\right]\right\}.$$

We express equation (19) by the following two expectations:

$$E_{floor} = E\left[1_{T\le t^0}\cdot e^{-rT}R_{floor}^{mort}\left(T\right)\right]$$

and

$$E_{upside} = E\left\{1_{T\le t^0}\cdot e^{-rT}\beta^{mort}\left[R_I^{mort}\left(T\right)-R_{th}^{mort}\left(T\right)\right]^+\right\}.$$

To calculate the above expectations we need to assume specific characteristics of the floor and threshold returns. We start with the simplest case, which seems also a very practical one, when both are constants:

$$R_{floor}^{mort}\left(T\right)=A_{floor}\quad\text{and}\quad R_{th}^{mort}\left(T\right)=A_{th}\ .$$

Then, for the constant floor (cf) we have:

$$E_{cf} = E\left[1_{T\le t^0}\cdot e^{-rT}A_{floor}\right] = A_{floor}\int_0^{t^0} e^{-rT}dF_T\ ,$$

where, for our examples, F_T follows Gompertz mortality time distribution function (see section 3.1.2).

Similarly, for the constant threshold (A_{th}) we write the following upside expectations:

$$E_{cup} = E\left\{1_{T\le t^0}\cdot e^{-rT}\beta^{mort}\left[R_I^{mort}\left(T\right)-A_{th}\right]^+\right\} = E\left\{1_{T\le t^0}\cdot e^{-rT}\beta^{mort}\left[e^{\left(\mu-0.5\sigma^2\right)T+\sigma\varepsilon\sqrt{T}}-A_{th}\right]^+\right\}.$$

For simplicity we omit the *mort* designation in the GBM expression above for $R_I^{mort}(T)$. The expectations can be formulated by the following integral:

$$E_{cup} = \beta^{mort} \int_0^{t^0} e^{-rT} dF_T \int 1_{\left\{\varepsilon > \frac{\ln(A_{th})-(\mu-0.5\sigma^2)T}{\sigma\sqrt{T}}\right\}} \left[e^{\left(\mu-0.5\sigma^2\right)T+\sigma\varepsilon\sqrt{T}} - A_{th} \right] dF_\varepsilon \, .$$

The inner integral has a solution that we use in the saving contract above. For convenience we define:

$$E_T = \int 1_{\left\{\varepsilon > \frac{\ln(A_{th})-(\mu-0.5\sigma^2)T}{\sigma\sqrt{T}}\right\}} \left[e^{\left(\mu-0.5\sigma^2\right)T+\sigma\varepsilon\sqrt{T}} - A_{th} \right] dF_\varepsilon$$

and:

$$d_T = \frac{\ln(A_{th}) - (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}} \quad .$$

Then, calculating the expectation integral for E_T we find:

$$E_T = e^{\left(\mu-0.5\sigma^2\right)T+0.5\sigma^2 T} \overline{\Phi}(d_T - \sigma\sqrt{T}) - A_{th}\overline{\Phi}(d_T) = e^{\mu T}\Phi(\sigma\sqrt{T} - d_T) - A_{th}\Phi(-d_T) \, .$$

Thus:

(20) $$E_{cup} = \beta^{mort} \int_0^{t^0} e^{-rT} \left[e^{\mu T}\Phi(\sigma\sqrt{T} - d_T) - A_{th}\Phi(-d_T) \right] dF_T \, .$$

Finally:

(21) $$PV_{FL\&S}\left(0\right) = PV_{FF\&T}\left(0\right) + B\left(1-k\right)E_{cup} \ ,$$

where $PV_{FF\&T}(0)$, the saving component is expressed by equation (17), and E_{cup}, the life insurance component is expressed by equation (20).

Lastly, to enhance the flexibility of this product we add a closed-form solution for a contract in which the payoff threshold of the life insurance component is the risk-free rate. This requires replacing E_{cup} in equation (21) by E_{rup} in equation (22):

(22) $$PV_{FL\&S}\left(0\right) = PV_{FF\&T}\left(0\right) + B\left(1-k\right)E_{rup} \, ,$$

where:

$$E_{rup} = \beta^{mort} \int_0^{t^0} e^{-rT} \left[e^{\mu T} \Phi(\sigma\sqrt{T} - d_T^r) - e^{rT} \Phi(-d_T^r) \right] dF_T \ ,$$

and

$$d_T^r = \frac{rT - (\mu - 0.5\sigma^2)T}{\sigma\sqrt{T}} \ .$$

The development of the E_{rup} expression is similar to that of E_{rup} after replacing A_{th} with e^{rT} .

3.3.2 Numerical examples for the life insurance and saving product

This section follows the same rationale and steps of 3.2.2. To simplify the exposition, without loss of generality, we use the same threshold and floor returns, and the same reference index for both the saving and the life insurance components. Table 3 presents the numerical values calculated using equations (21) and (22) for the life insurance and saving product, assuming the reference index returns follow a GBM process. Constant threshold in panels A-C using equation (21) and risk-free rate threshold in panels D-F using equation (22). Similar to tables 1 and 2, each line shows results for a specified interest-rate and columns are by age and contract period in years. We limit the presentation to only a few examples for the product parameters and show results for β_{mort} = 1.

Table 3 about here

To calculate $PV_{FL\&S}(0)$, the present value for life insurance and saving product, assuming the reference index returns follow a JD process, we use MC simulations of equation (8). Following the same procedure of section 3.2.2 (100 observations, each of 10,000 paths) results in a standard deviation of about 0.35% of the average value, for *age* = 40 years, R_{th} = 1, R_{floor} = 1, β_{saving} = 0.5, and β_{mort} = 1. Hence, we decide to use 100,000 paths per MC estimation, reducing the standard deviation to about 0.11% of the average value. Table 4 presents the numerical values for the life insurance and saving product, assuming the reference index returns follow a JD process.

Table 4 about here

3.4 The disability insurance and saving product example

In this section we calculate numerical examples for $PV_{FD\&S}(0)$, the value of the disability insurance and saving product, using equation (10). For this product we do not develop a closed-form solution and we use MC simulations only.

Here again we decide to use 100,000 paths per MC estimation, estimating the standard deviation at about 0.03% and 0.1% of the average value, for the GBM and JD processes respectively. These values are based on the same procedure we use in section 3.2.2 (100 observations, each of 10,000 paths), which results in a standard deviation of about 0.1% (0.34%) of the average value, for the GBM (JD) process, *age* = 40 years, R_{th} = 1, R_{floor} = 1, β_{saving} = 0.5, and β_{disab} = 1. Tables 5 and 6 present the numerical values for the life insurance and saving product, assuming the reference index returns follow a GBM and JD processes, respectively. The examples and parameter choice parallel those of section 3.3 and we present results for β_{disab} = 1 only.

Tables 5+6 about here

4. Discussion

We start with a few general observations prior to a more detailed discussion. As one could expect, the first product, which is a pure saving product, has a lower value than the other two products which provide payoffs upon death or disability event in addition to the saving component. The main novelty of the first product compared to common structured products is that its value is lower because its eventual payment is contingent on the insured being alive by the end of the contract period. The differences between the products' values vary widely, depending on the insured age, the contract horizon, and the other contract parameters. The risk-free rate has a strong influence while the results for the GBM and for the JD processes are quite similar.

The effects of the floor (R_{floor}), the threshold (R_{th}), and the participation in the upside returns (β) are quite trivial and monotone. An increase in the floor and the participation increases the value of the product whereas an increase in the threshold decreases the value of the product. The effect of the floor can be seen when comparing panels A to C and D to F in any of the tables. Increasing the floor increases the minimal future payoff for the investor and thus obviously increases the cost for the issuer and the value of the product. Increasing the threshold reduces the potential positive spread between the index and the threshold, reducing the expected upside of the product and thus its value. This effect can be seen when comparing panels A to D, B to E, and C to F in any of the tables, where D, E, and F use the risk-free rate for the threshold and A, B, and C use R_{th} = 1 which is lower than the returns on the risk free asset (we assume positive risk-free rates throughout). The participation (β) is a multiplier of the upside potential returns and thus increasing it increases the expected payoffs and the value of the product as can been seen when comparing panels A to B or D to E in any of the tables. For simplicity we hold β_{mort} = 1 and β_{disab} = 1 constant, for the life insurance and disability insurance components respectively.

The effect of age, horizon, and interest rate is less trivial than that of the floor, threshold, and participation. For the case of the first (saving) product the age effect is simple. The older the insured is, the higher are the chances of mortality prior to the end of the contract, hence the lower is the value of the investment contract. This is evident in all the tables, the value for age 30 is higher by 1.8, 4.8, and 18.2%, approximately, relative to age 40, for horizons of 5, 10, and 20 years respectively, unrelated to the interest rate or the specifications of the first (saving) product. For the second and third products, which include life and disability insurance components respectively, an increased age increases the value of these insurance components while, similar to the saving product, it decreases the value of the saving component. The net effect of age depends on the specifics of the contract, the interest rate, and the age of the insured. For brevity we show examples for ages of 30 and 40 years only. In our examples the sign of the net effect is mixed. In the majority of our panels for the second (life insurance and saving) product the net effect of increasing the insured age is positive, often a fraction of a percent and reaching a few percent for the 20-year contracts.

Ceteris paribus, increasing the interest rate decreases the value of all three products (see all tables) because it is assumed that future payoffs are either not related to the risk free rate (panels A-C of all tables), or it affects the threshold and thus decreases also the future payoffs and not only their present value (panels D-F of all tables). However, it is important to note here two key matters: (i) for clarity of the exposition and analysis we use fixed models for the index and assume that the GBM and JD models in this paper are not affected by the prevailing interest rate, an assumption that often does not hold in the real market; (ii) for brevity we use only constant floor in our examples. Yet, when the floor is set at the risk-free return, an increase of the interest rate would increase the future payoffs, hence would affect the present value of the product positively, in the opposite direction to that of the discount rate. The net result would depend on the other parameters and variables of the contract and the scenario.

The two index models, the GBM and the JD processes are dissimilar, particularly regarding tail events. Yet the results for the two processes are not far apart (odd numbered tables are for the GBM process and even numbered tables for the JD process). We attribute this similarity to the nature of the contracts, they are usually long term and thus the effect of jumps in the JD process smooths out over the long period of the contract, especially when averaged over multiple paths. This is a proper result for an insurer with many customers, issuing many contracts at multiple times. When a single contract is issued the nature of the problem changes materially, its discussion is beyond the scope of this paper, and we think that it is also a particular riddle that does not fit the scheme of this paper which addresses the mass customization challenge. A practical outcome of the comparison between the two processes is that for a valuation estimate one could use the GBM process, with its relatively easy calculation task, and often does not need to apply the MC simulation for the JD process with its much higher computational burden.

5. Conclusions

The world experiences rapid changes in recent decades and it appears inevitable that this would continue at an accelerating pace in the coming decades. In this fast-changing environment, the insurance industry faces new challenges of changing demographics, volatile financial markets, disruptive technology, and intensifying competition. Yet, these challenges also bring about new opportunities for insurers and their customers.

Furthermore, in recent decades we have seen outstanding progress in financial engineering modeling and hedging. The global financial markets provide a continuously widening spectrum of underlying assets for investments and hedging purposes. Structured products have evolved over the years and are currently considered quite mature investment vehicles. However, they generally do-not embed life and health contingencies. This paper presents foundations for a wide spectrum of saving and insurance hybrid and flexible products. We demonstrate our idea via three families of structured contracts, describing their general construction and characteristics, then providing a few examples of closed-form solutions when the underlying reference assets follow GBM processes, and finally providing a broad sample of numerical solutions for all three families, using both GBM and JD process examples. It seems inevitable that insurers would be compelled to offer such products to their clientele in response to their customers' demand (existing and potential customers) and as a strategic initiative or a response to competitive pressures in their market. Nowadays, the technology for mass-customization of such contracts is available and widely accessible to the firms and their clientele. Our work demonstrates that the models for such insurance structured contracts are tractable and flexible, enabling the immediate application of the flexible hybrid product idea on a large scale.

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Tables and Figures

Table 1 - Saving product numerical examples using GBM process

The table presents the numerical values calculated using equation (17) for the saving product assuming the reference index returns follow a GBM process. Age and contract period are specified in years. The interest rate is per annum, continuously compounded.

Panel A: R_{th} = 1, R_{floor} = 1, β = 0.5

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0616	1.1610	1.4022	1.0428	1.1082	1.1864
	0.03	0.9605	0.9505	0.9399	0.9436	0.9073	0.7953
	0.05	0.8691	0.7782	0.6300	0.8538	0.7429	0.5331

Panel B: R_{th} = 1, R_{floor} = 1, β = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2295	1.4870	2.1040	1.2078	1.4195	1.7802
	0.03	1.1125	1.2175	1.4104	1.0929	1.1622	1.1933
	0.05	1.0066	0.9968	0.9454	0.9889	0.9515	0.7999

Panel C: R_{th} = 1, R_{floor} = 0.9, β = 0.5

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9722	1.0775	1.3322	0.9550	1.0285	1.1271
	0.03	0.8797	0.8822	0.8930	0.8642	0.8421	0.7555
	0.05	0.7960	0.7223	0.5986	0.7819	0.6894	0.5065

Panel D: R_{th} = risk-free, R_{floor} = 1, β = 0.5

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0463	1.1290	1.3399	1.0278	1.0777	1.1337
	0.03	0.9202	0.8718	0.8030	0.9040	0.8322	0.6794
	0.05	0.8106	0.6729	0.4705	0.7963	0.6423	0.3981

Panel E: R_{th} = risk-free, R_{floor} = 1, β = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.1990	1.4232	1.9795	1.1778	1.3585	1.6749
	0.03	1.0319	1.0600	1.1364	1.0137	1.0118	0.9615
	0.05	0.8896	0.7861	0.6263	0.8739	0.7504	0.5299

Panel F: R_{th} = risk-free, R_{floor} = 0.9, β = 0.5

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9570	1.0455	1.2699	0.9401	0.9980	1.0745
	0.03	0.8394	0.8034	0.7560	0.8246	0.7669	0.6397
	0.05	0.7375	0.6169	0.4390	0.7245	0.5889	0.3714

Table 2 - Saving product numerical examples using JD process

The table presents the numerical values calculated using MC simulations of equation (5) for the saving product, assuming the reference index returns follow a JD process. Age and contract period are specified in years. The interest rate is per annum, continuously compounded.

Panel A: $R_{th} = 1$, $R_{floor} = 1$, $\beta = 0.5$

Age →		30			40		
Contract Period →		5	10	20	5	10	20
Interest Rate	0.01	1.0677	1.1706	1.3820	1.0615	1.1394	1.1665
	0.03	0.9669	0.9600	0.9279	0.9619	0.9382	0.7822
	0.05	0.8736	0.7873	0.6275	0.8702	0.7714	0.5369

Panel B: $R_{th} = 1$, $R_{floor} = 1$, $\beta = 1$

Age →		30			40		
Contract Period →		5	10	20	5	10	20
Interest Rate	0.01	1.2319	1.4913	2.0738	1.2242	1.4460	1.7438
	0.03	1.1148	1.2205	1.3913	1.1088	1.1848	1.1760
	0.05	1.0083	0.9993	0.9370	1.0027	0.9785	0.8009

Panel C: $R_{th} = 1$, $R_{floor} = 0.9$, $\beta = 0.5$

Age →		30			40		
Contract Period →		5	10	20	5	10	20
Interest Rate	0.01	0.9767	1.0849	1.3034	0.9702	1.0562	1.0802
	0.03	0.8848	0.8895	0.8732	0.8793	0.8664	0.7303
	0.05	0.7997	0.7299	0.5899	0.7962	0.7123	0.4945

Panel D: R_{th} = risk-free, R_{floor} = 1, β = 0.5

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0525	1.1388	1.3226	1.0459	1.1132	1.1281
	0.03	0.9258	0.8826	0.8127	0.9228	0.8695	0.7183
	0.05	0.8167	0.6837	0.4916	0.8152	0.6794	0.4598

Panel E: R_{th} = risk-free, R_{floor} = 1, β = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2015	1.4306	1.9570	1.1945	1.3875	1.6735
	0.03	1.0366	1.0658	1.1411	1.0295	1.0432	0.9917
	0.05	0.8914	0.7943	0.6410	0.8894	0.7860	0.5835

Panel F: R_{th} = risk-free, R_{floor} = 0.9, β = 0.5

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9618	1.0544	1.2457	0.9571	1.0293	1.0456
	0.03	0.8450	0.8116	0.7618	0.8397	0.7962	0.6617
	0.05	0.7420	0.6269	0.4537	0.7411	0.6207	0.4195

Table 3 - Life insurance and saving product examples using GBM process

The table presents the numerical values for the life insurance and saving product assuming the reference index returns follow a GBM process. Panels A-C use equation (21) and panels D-F equation (22). Age and contract period are specified in years. The interest rate is per annum, continuously compounded.

Panel A: R_{th} = 1, R_{floor} = 1, β_{saving} = 0.5, β_{mort} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0741	1.1981	1.5785	1.0750	1.2021	1.6040
	0.03	0.9724	0.9834	1.0738	0.9740	0.9906	1.1139
	0.05	0.8803	0.8075	0.7328	0.8826	0.8170	0.7788

Panel B: R_{th} = 1, R_{floor} = 1, β_{saving} = 1, β_{mort} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2420	1.5242	2.2803	1.2399	1.5134	2.1978
	0.03	1.1243	1.2504	1.5443	1.1233	1.2455	1.5119
	0.05	1.0178	1.0260	1.0482	1.0176	1.0256	1.0456

Panel C: R_{th} = 1, R_{floor} = 0.9, , β_{saving} = 0.5, β_{mort} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9837	1.1120	1.5002	0.9845	1.1159	1.5247
	0.03	0.8905	0.9127	1.0204	0.8920	0.9195	1.0585
	0.05	0.8062	0.7494	0.6962	0.8083	0.7583	0.7397

Panel D: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 0.5, β_{mort} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0586	1.1651	1.5076	1.0595	1.1690	1.5310
	0.03	0.9316	0.9020	0.9167	0.9331	0.9086	0.9507
	0.05	0.8211	0.6983	0.5481	0.8232	0.7069	0.5845

Panel E: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 1, β_{mort} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2113	1.4592	2.1472	1.2095	1.4498	2.0721
	0.03	1.0432	1.0902	1.2502	1.0428	1.0883	1.2328
	0.05	0.9001	0.8116	0.7038	0.9008	0.8150	0.7163

Panel F: R_{th} = risk-free, R_{floor} = 0.9, , β_{saving} = 0.5, β_{mort} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9682	1.0790	1.4292	0.9691	1.0828	1.4518
	0.03	0.8497	0.8313	0.8633	0.8512	0.8375	0.8952
	0.05	0.7470	0.6403	0.5114	0.7490	0.6482	0.5454

Table 4 - Life insurance and saving product numerical examples using JD process

The table presents the numerical values calculated using MC simulations of equation (8) for the life insurance and saving product assuming the reference index returns follow a JD process. Age and contract period are specified in years. The interest rate is per annum, continuously compounded.

Panel A: R_{th} = 1, R_{floor} = 1, β_{saving} = 0.5, β_{mort} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0706	1.1921	1.5665	1.0718	1.1952	1.5909
	0.03	0.9700	0.9790	1.0635	0.9716	0.9848	1.1021
	0.05	0.8774	0.8024	0.7279	0.8807	0.8136	0.7720

Panel B: R_{th} = 1, R_{floor} = 1, β_{saving} = 1, β_{mort} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2391	1.5180	2.2646	1.2340	1.5041	2.1665
	0.03	1.1213	1.2461	1.5297	1.1187	1.2369	1.4968
	0.05	1.0136	1.0229	1.0340	1.0129	1.0181	1.0333

Panel C: R_{th} = 1, R_{floor} = 0.9, , β_{saving} = 0.5, β_{mort} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9804	1.1050	1.4869	0.9822	1.1080	1.5039
	0.03	0.8880	0.9076	1.0113	0.8899	0.9167	1.0458
	0.05	0.8045	0.7442	0.6865	0.8066	0.7557	0.7354

Panel D: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 0.5, β_{mort} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0556	1.1582	1.4903	1.0564	1.1660	1.5122
	0.03	0.9299	0.8975	0.9039	0.9306	0.9056	0.9372
	0.05	0.8187	0.6936	0.5400	0.8209	0.7036	0.5749

Panel E: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 1, β_{mort} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2053	1.4479	2.1102	1.2022	1.4427	2.0501
	0.03	1.0382	1.0828	1.2381	1.0366	1.0754	1.2106
	0.05	0.8951	0.8023	0.6891	0.8964	0.8073	0.7055

Panel F: R_{th} = risk-free, R_{floor} = 0.9, , β_{saving} = 0.5, β_{mort} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9661	1.0728	1.4104	0.9663	1.0769	1.4319
	0.03	0.8480	0.8270	0.8505	0.8490	0.8363	0.8829
	0.05	0.7450	0.6360	0.5055	0.7469	0.6442	0.5386

Table 5 - Disability insurance and saving product examples using GBM process

The table presents the numerical values calculated using MC simulations of equation (10) for the disability insurance and saving product assuming the reference index returns follow a GBM process. Age and contract period are specified in years. The interest rate is per annum, continuously compounded.

Panel A: $R_{\text{th}} = 1$, $R_{\text{floor}} = 1$, $\beta_{saving} = 0.5$, $\beta_{disab} = 1$

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0734	1.1947	1.5567	1.0732	1.1943	1.5554
	0.03	0.9712	0.9786	1.0438	0.9710	0.9784	1.0422
	0.05	0.8787	0.8013	0.7002	0.8789	0.8015	0.7006

Panel B: $R_{\text{th}} = 1$, $R_{\text{floor}} = 1$, $\beta_{saving} = 1$, $\beta_{disab} = 1$

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2423	1.5300	2.3330	1.2429	1.5300	2.3200
	0.03	1.1254	1.2533	1.5649	1.1246	1.2528	1.5566
	0.05	1.0181	1.0265	1.0498	1.0179	1.0262	1.0446

Panel C: $R_{\text{th}} = 1$, $R_{\text{floor}} = 0.9$, , $\beta_{saving} = 0.5$, $\beta_{disab} = 1$

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9831	1.1097	1.4803	0.9831	1.1087	1.4764
	0.03	0.8889	0.9079	0.9919	0.8897	0.9085	0.9916
	0.05	0.8052	0.7434	0.6650	0.8049	0.7436	0.6659

Panel D: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 0.5, β_{disab} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0584	1.1627	1.4870	1.0580	1.1629	1.4845
	0.03	0.9306	0.8980	0.8910	0.9304	0.8972	0.8919
	0.05	0.8195	0.6931	0.5223	0.8197	0.6928	0.5231

Panel E: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 1, β_{disab} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2123	1.4656	2.1941	1.2123	1.4639	2.1840
	0.03	1.0437	1.0913	1.2607	1.0427	1.0911	1.2558
	0.05	0.8998	0.8103	0.6959	0.8988	0.8097	0.6939

Panel F: R_{th} = risk-free, R_{floor} = 0.9, , β_{saving} = 0.5, β_{disab} = 1

Age →		30			40		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9678	1.0763	1.4112	0.9676	1.0761	1.4072
	0.03	0.8489	0.8270	0.8394	0.8486	0.8272	0.8391
	0.05	0.7457	0.6352	0.4875	0.7456	0.6351	0.4884

Table 6 - Disability insurance and saving product examples using JD process

The table presents the numerical values calculated using MC simulations of equation (10) for the disability insurance and saving product assuming the reference index returns follow a JD process. Age and contract period are specified in years. The interest rate is per annum, continuously compounded.

Panel A: R_{th} = 1, R_{floor} = 1, β_{saving} = 0.5, β_{disab} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0707	1.1892	1.5469	1.0691	1.1906	1.5388
	0.03	0.9702	0.9745	1.0333	0.9694	0.9748	1.0275
	0.05	0.8775	0.7970	0.6914	0.8773	0.7974	0.6920

Panel B: R_{th} = 1, R_{floor} = 1, β_{saving} = 1, β_{disab} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2393	1.5205	2.3053	1.2394	1.5200	2.2866
	0.03	1.1203	1.2440	1.5480	1.1189	1.2444	1.5383
	0.05	1.0154	1.0171	1.0329	1.0134	1.0194	1.0346

Panel C: R_{th} = 1, R_{floor} = 0.9, , β_{saving} = 0.5, β_{disab} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9809	1.1029	1.4625	0.9810	1.1064	1.4670
	0.03	0.8879	0.9052	0.9807	0.8874	0.9039	0.9810
	0.05	0.8027	0.7403	0.6585	0.8027	0.7396	0.6585

Panel D: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 0.5, β_{disab} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.0546	1.1571	1.4710	1.0556	1.1572	1.4675
	0.03	0.9287	0.8929	0.8808	0.9284	0.8944	0.8842
	0.05	0.8180	0.6885	0.5157	0.8170	0.6885	0.5174

Panel E: R_{th} = risk-free, R_{floor} = 1, β_{saving} = 1, β_{disab} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	1.2073	1.4560	2.1622	1.2065	1.4542	2.1522
	0.03	1.0393	1.0828	1.2320	1.0388	1.0822	1.2340
	0.05	0.8943	0.8020	0.6840	0.8959	0.8025	0.6804

Panel F: R_{th} = risk-free, R_{floor} = 0.9, , β_{saving} = 0.5, β_{disab} = 1

Age →		**30**			**40**		
Contract Period →		**5**	**10**	**20**	**5**	**10**	**20**
Interest Rate	0.01	0.9642	1.0710	1.3931	0.9643	1.0711	1.3936
	0.03	0.8463	0.8217	0.8296	0.8466	0.8219	0.8335
	0.05	0.7432	0.6315	0.4814	0.7433	0.6317	0.4820